Cambridge IGCSE™

CANDIDATE NAME	

CENTRE NUMBER			CANDIDATE NUMBER	

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/52**

Paper 5 Investigation (Core) **February/March 2021**

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

[Turn over

CONSECUTIVE NUMBERS (36 marks)

This task is about what happens when consecutive numbers are changed from positive to negative and added.

Consecutive numbers are sequences of integers which increase by 1 from term to term.

<u>Examples</u>   0, 1, 2, 3, 4 or 5, 6, 7 or 46, 47, 48, 49, 50, 51 or 3, 4, 5, ..., 120.

In this investigation use this **method** throughout.

- Add the positive consecutive numbers.
- Find all the possible additions and totals when you make **one** of the numbers negative.
- Find all the possible additions and totals when you make **two** of the numbers negative.
- Continue in this way until all the numbers are negative.

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1   1, 2 is a sequence of two consecutive numbers.

    (a) **(i)**   Complete the table using the method with 1 and 2 to find all the possible totals.

	Addition				Total
All positive	1	+	2	=	3
One negative	−1	+	2	=	....
	1	+	−2	=	....
All negative	−1	+	−2	=	−3

[1]

        (ii)   Using the consecutive numbers 1 and 2 the highest total is 3 and the lowest total is −3.

            You cannot make all the integers between the highest total and the lowest total using the method.

            Write down all the integers between 3 and −3 that **cannot** be made using 1 and 2.
            Remember: 0 is an integer.

            .... [1]

(b) (i) Complete the table using the method with the consecutive numbers 2 and 3.

	Addition				Total
All positive	2	+	3	=	5
One negative	−2	+	....	=	1
	2	+	....	=	....
All negative	−2	+	−3	=	....

[2]

(ii) Using the table in **part (i)**, complete these statements.

The highest total is 5 and the lowest total is ....

The number of integers between the highest total and the lowest total that **cannot** be made is

.... [2]

(c) (i) Complete the table using the method with two consecutive numbers.

	Addition				Total
All positive	....	+	....	=	15
One negative	....	+	....	=	....
	....	+	....	=	....
All negative	....	+	....	=	−15

[2]

(ii) Find the number of integers between 15 and −15 that **cannot** be made using these consecutive numbers.

.... [1]

2   *a* and *a* + 1 are two consecutive numbers.

   (a)   Find expressions for the four totals that **can** be made using *a* and *a* + 1.
        Give each expression in its simplest form.

.... , ... , ... , ... [3]

   (b)   An expression for the number of integers between the highest total and the lowest total that **cannot**
        be made using *a* and *a* + 1 is $4a - 1$.

        Show that this gives the correct number when $a = 10$.

[4]

3 **(a)** There are now three consecutive numbers.

(i) Complete the table using the method with the consecutive numbers 3, 4 and 5.

	Addition						Total
All positive	3	+	4	+	5	=	12
One negative	−3	+	4	+	5	=	6
	....	+	−4	+	....	=	4
	....	+	....	+	....	=	2
Two negative	−3	+	−4	+	....	=	−2
	−3	+	....	+	−5	=	....
	....	+	....	+	....	=	....
All negative	−3	+	−4	+	−5	=	−12

[2]

(ii) Find the number of integers that **cannot** be made between 12 and −12.

.... [2]

(b) There are now four consecutive numbers.

Complete the table using the method and the consecutive numbers 3, 4, 5 and 6.

	Addition								Total
All positive	3	+	4	+	5	+	6	=	18
One negative	−3	+	4	+	5	+	6	=	12
	....	+	−4	+	5	+	6	=	10
	3	+	4	+	−5	+	6	=	8
	....	+	....	+	....	+	....	=	....
Two negative	−3	+	−4	+	5	+	6	=	....
	−3	+	4	+	....	+	6	=	2
	−3	+	4	+	5	+	−6	=	0
	3	+	−4	+	....	+	6	=	0
	3	+	−4	+	....	+	−6	=	....
	3	+	4	+	−5	+	−6	=	−4
Three negative	−3	+	−4	+	....	+	....	=	....
	−3	+	−4	+	....	+	....	=	−8
	−3	+	4	+	−5	+	....	=	−10
	3	+	....	+	....	+	....	=	−12
All negative	−3	+	−4	+	−5	+	−6	=	−18

[3]

TURN OVER FOR QUESTION 4

4 **(a)** There are 16 additions in the table on **page 6**.

Complete the table below.
Use **Question 1** and **Question 3** to help you.

Number of consecutive numbers	Number of additions		
2	....	=	....
3	....	=	2^3
4	16	=	2^4
5	32	=	....
n			....

[2]

(b) Complete this table.
Use **Question 2(a)** to help you.

Number of consecutive numbers	Consecutive numbers	Expression for the highest total in terms of a
2	$a, a + 1$	
3		$3a + 3$
4		
5	$a, a + 1, a + 2, a + 3, a + 4$	
n		$\ldots\ldots\ldots + \dfrac{n(n-1)}{2}$

[5]

(c) Anna uses this method to work out the number of integers that **cannot** be made.

- Use **Question 4(b)** to find the highest total.

- Find the number of integers from the highest total to the lowest total.

- Use **Question 4(a)** to find the number of additions.

- Subtract the number of additions from the number of integers.

<u>Example</u>
There are three consecutive numbers.
The first number is 4.

The highest total is $3a + 3 = 3 \times 4 + 3 = 15$.
The number of integers from 15 to -15 is 31.
The number of additions is 2^3.
The number of integers that **cannot** be made is $31 - 2^3 = 23$.

(i) There are two consecutive numbers.

Use Anna's method to find the number of integers that **cannot** be made when the first number is 9.

.... [3]

(ii) Anna uses her method to find the number of integers that **cannot** be made with the three consecutive numbers 1, 2 and 3.
Her method gives the answer 5.

Explain why her method gives the wrong answer.

.... [3]

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